VKSU

Rate of Energy Transmission

The Kinetic energy dK attached with a string element of mass dm is : $dK = \frac{1}{2} dm\, u^2$ — (1)

where u = Transverse speed of the oscillating string element. we know That

$y(x, t) - y_m \sin(Kx - \omega t)$.

To Calculate The Value of u, we differentiate above equn w.r.t. time, while holding x constant:

$u = \partial y / \partial t = -\omega y_m \cos(Kx - \omega t)$ — (2)

By Putting $dm = \mu dx$ in equn (1), we get

$dK = \frac{1}{2}(\mu dK)(-\omega y_m)^2 \cos^2(Kx - \omega t)$ — (3)

on dividing above equn (3), by dt, we get The rate where The Kinetic energy Passes Through a string element and Therefore Kinetic energy is Carried along The wave. The ratio dx/dt appearing on The right hand Side of equn (3) is The wave speed v, So we get:

$$\frac{dK}{dt} = \frac{1}{2} \mu v \omega^2 y_m^2 \cos^2(Kx - \omega t) \quad \text{— (4)}$$

The average rate at which Kinetic energy is transported is given by

$$\left(\frac{dK}{dt}\right)_{avg} = \frac{1}{2} \mu v \omega^2 y_m^2 \left[\cos^2(Kx - \omega t)\right]_{avg}$$

$$= \frac{1}{4} \mu v \omega^2 y_m^2 \quad \text{— (5)}$$

In The above equn The average over an integer number of wavelength is taken and we have

used the fact that the average value of the square of a cosine function over an integer number of periods is $1/2$.
The elastic potential energy also moves with the wave, and at the same average rate given by equn (5). However, in an oscillating system such as a pendulum or a spring-block system, the average kinetic energy is equal to the average potential energy.
The average power is the power at which both types of energy are transmitted through the wave and it can be represented as:

$$P_{avg} = 2\left(\frac{dK}{dt}\right)_{avg} \quad \text{---} \ (6)$$

From equn (5), we have:

$$P_{avg} = \frac{1}{2} \mu v \omega^2 y_m^2 \quad (\text{Average Power})$$

where factors μ and v are based upon the material and tension of the string. The factors ω and y_m is based upon the process that generates the wave.
The average power of a wave depends on the square of its amplitude as well as the square of its angular frequency and we get a general result that holds true for all types of waves.

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Ex Calculate what average rate does the wave transport energy if a string has linear density $\mu = 525\ g/m$ and tension $\Upsilon = 45N$. We send a sinusoidal wave with frequency $f = 120\ Hz$ and amplitude $y_m = 8.5\ mm$ along the string.

Sol: At first we need to calculate the angular frequency ω and the wave speed v.

$$\omega = 2\pi f = (2\pi)(120\ Hz) = 754\ rad/s$$

we have $$v = \sqrt{\frac{\Upsilon}{\mu}} = \sqrt{\frac{45N}{0.525\ Kg/m}} = 9.26\ m/s.$$

According to equn (7), we get

$$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2$$

on substituting all the values in above equn, we have

$$P_{avg} = \left(\frac{1}{2}\right)(0.525)(9.26)(754)^2(0.0085)^2$$

$$= \underline{100\ W.}$$

Phase velocity / Wave velocity

Phase or wave velocity is defined as the velocity with which are plane of equal phase (crest or trough) travels through the medium.

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The equation of Phase Progressive wave is given by

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) = a \sin(\omega t - x) \quad \text{--- (1)}$$

where,

$\omega = 2\pi n$ and $K = \frac{2\pi}{\lambda}$;

$$\frac{\omega}{K} = \frac{2\pi n}{2\pi/\lambda} = n\lambda = V$$

This V is defined as The Phase velocity or The wave velocity. Thus "Phase velocity is defined as The velocity with That a Plane Progressive wave front travels forward and has a constant Phase given by $(\omega t - x)$

Therefore,

$(\omega t - x) = $ a constant — (2)

Now differentiate equn (2) w.r.t. t we get

$$\omega - K \frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = \frac{\omega}{K} = V \text{ (Phase velocity)}$$